

Roll No. ....

**DD-2808**

**M. A./M. Sc. (Final) EXAMINATION, 2020**

**MATHEMATICS**

**(Optional)**

**Paper Third (i)**

**(Graph Theory)**

*Time : Three Hours*

*Maximum Marks : 100*

**Note :** All questions are compulsory. Solve any *two* parts from each question. All questions carry equal marks.

**Unit—I**

1. (a) Prove that the sum of any two cuts of a graph  $G$  is also a cut of  $G$ .
- (b) Prove that if a graph  $G$  is contractible to a graph  $H$  and  $\Delta(H) \leq 3$ , then  $G$  has a subgraph Homeomorphic from  $H$ .
- (c) Define the following terms :
  - (i) Cycle space and Bond space
  - (ii) Determinant and the spectrum

## Unit—II

2. (a) Show that for a graph  $G$  :
- $$\Delta(G) \leq \psi'(G) \leq \Delta G + 1$$
- (b) Prove that every planar graph is  $K$ -vector colourable iff every plane graph is  $K$ -face colourable.
- (c) Define the following terms :
- (i) The Resenfeld numbers
  - (ii) Girth and chromatic number

## Unit—III

3. (a) Prove that for any  $S \geq 2$  :
- $$R(S, S) \geq 2^{S/2}$$
- (b) Prove that a graph  $G$  is a permutation graph iff  $G$  and  $\bar{G}$  are comparability graphs.
- (c) Define the following terms :
- (i) SPGC and split graphs
  - (ii) Ramsey numbers and Ramsey graphs

## Unit—IV

4. (a) Prove that if the eigen values of a digraph are all distinct, then  $T(\Delta)$  is abelian.
- (b) Prove that every group is isomorphic to the automorphism group of some graph.
- (c) Define the following terms :
- (i) Co-chromatic graphs and chromatically unique graphs
  - (ii) The chromatic polynomial and graph enumeration

## Unit—V

5. (a) State and prove Konig's theorem.
- (b) State and prove Menger's theorem.
- (c) Define the following terms :
- (i) Digraphs and types of connectedness
  - (ii) Networks and degree sequences